Eur. Phys. J. C 8, 81–90 (1999)
DOI 10.1007/s100529901060

THE EUROPEAN PHYSICAL JOURNAL C

Semileptonic and rare B-meson transitions in a QCD relativistic potential model

P. Colangelo[1], F. De Fazio[1], M. Ladisa[1,2], G. Nardulli[1,2], P. Santorelli[1,2], A. Tricarico[2]

[1] Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy
[2] Dipartimento di Fisica dell'Università di Bari, Italy

Received: 14 September 1998 / Published online: 11 March 1999

Abstract. Using a QCD relativistic potential model, previously applied to the calculation of the heavy meson leptonic constants, we evaluate the form factors governing the exclusive decays $B \to \rho\ell\nu$, $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$. In our approach the heavy meson is described as a $Q\bar{q}$ bound state, whose wave function is solution of the relativistic Salpeter equation, with an instantaneous potential displaying Coulombic behaviour at small distances and linear behaviour at large distances. The light vector meson is described by using a vector current interpolating field, according to the Vector Meson Dominance assumption. A Pauli-Villars regularized propagator is assumed for the quarks not constituting the heavy meson. Our procedure allows to avoid the description of the light meson in terms of wave function and constituent quarks, and consequently the problem of boosting the light meson wave function. Assuming as an input the experimental results on $B \to K^*\gamma$, we evaluate all the form factors describing the $B \to \rho, K^*$ semileptonic and rare transitions. The overall comparison with the data, whenever available, is satisfactory.

1 Introduction

B meson decays play a central role in particle physics, as witnessed by the considerable amount of experimental data collected, mainly at CESR, LEP, Tevatron and SLAC accelerators. More of all, the importance of B decay processes is related to the results which will come in the near future from BaBar and Belle experiments at the dedicated SLAC and KEK B-facilities, and from the LHC-B experiment at CERN, whose main goal is the analysis of CP violation in the B system [1]. Consequently, theoretical efforts are greatly projected towards the determination of methods to extract, from the data, the elements of the Cabibbo-Kobayashi-Maskawa matrix (CKM), since the complex nature of this matrix is the source of CP violation within the Standard Model (SM). From this point of view, heavy-to-light decays are of prime interest, since $b \to u$ transitions offer the possibility of determining the poorly known matrix element V_{ub}, while $b \to s$ processes, forbidden at tree level in SM, give access to V_{ts} and, in addition, represent a powerful tool to investigate possible effects of physics beyond SM.

In this paper we address both $b \to u$ and $b \to s$-induced decay channels, and in particular we analyze the exclusive semileptonic and rare B decays to a nonstrange ρ and strange K^* vector mesons.

The branching ratio of the semileptonic decay $B \to \rho\ell\nu$ has been recently measured by the CLEO collaboration [2]:

$$\mathcal{B}(B^0 \to \rho^-\ell^+\nu) = (2.5 \pm 0.4^{+0.5}_{-0.7} \pm 0.5) \times 10^{-4} \quad . \quad (1)$$

From the experimental viewpoint, due to the overwhelming $b \to c$ transitions, such decay is not easily accessible, and one has to select leptons in a high momentum range which can be reached in the $b \to u\ell\nu$ transitions, but not in the $b \to c\ell\nu$ process. In addition to such experimental difficulty, in order to extract V_{ub} from the data one has to deal with the theoretical uncertainty arising from the evaluation of the hadronic $B \to \rho$ matrix element.

The analysis of the rare decays $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$ presents analogous uncertainties. Experimental data already exist for both the inclusive $b \to s\gamma$ and the exclusive decays with a real photon in the final state:

$$\mathcal{B}(b \to s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4} [3] \quad (2)$$

$$\mathcal{B}(\bar{B}^0 \to K^{*0}\gamma) = (4.0 \pm 1.7 \pm 0.8) \times 10^{-5} [4] \quad (3)$$
$$\mathcal{B}(B^- \to K^{*-}\gamma) = (5.7 \pm 3.1 \pm 1.1) \times 10^{-5} \text{ .} [4]$$

These results constrain the parameters featuring various new physics models, since rare B decays are particularly sensitive to effects beyond SM, but also in this case the interpretation depends on the reliable evaluation of the relevant hadronic $B \to K^*$ matrix elements.

To deal with such non perturbative quantities of the B physics few theoretical approaches are available so far. Directly related to the QCD description of strong interactions are Lattice QCD and QCD Sum Rules. Lattice QCD, based on the procedure of discretizing the space-time, allows a numerical evaluation of the hadronic matrix elements [5]. QCD Sum Rules, in the versions of three-point Sum Rules (SR) and Light Cone Sum Rules (LCSR) are based on fundamental properties of quark current correlators, such as the analiticity and the possibility of expanding at short-distances or on the light-cone. [6]. The above approaches have their own advantages and drawbacks, in particular systematic uncertainties related to the quenched approximation for Lattice QCD, and errors induced by the truncation of the Operator Product Expansion for the Sum Rules. Therefore, it is worth looking for other approaches that, while being less fundamental, present nevertheless the advantage of computational simplicity and offer at the same time a sufficiently deep physical insight.

To study the transitions between the B meson and the light vector mesons ρ and K^* and to compute the relevant hadronic matrix elements we employ in this paper a QCD relativistic potential model. The main aspect of the model is that the heavy meson is described as a $Q\overline{q}$ bound state; the wave function is obtained by solving the relativistic Salpeter equation [7], with an instantaneous potential displaying Coulombic behaviour at short distances and linear (confining) behaviour at large distances. The light vector meson is described by using a vector current interpolating field, following the Vector Meson Dominance Model. Finally, a Pauli-Villars regularized propagator is assumed for the quarks not constituting the heavy meson. Since we do not describe the light meson in terms of wave function and constituent quarks, we can avoid the problem of boosting the wave function, a point which is, in general, a source of considerable ambiguity.

Our approach represents an extension to the problem of heavy-light transitions of the work in [8] where the spectrum of $\bar{q}Q$ mesons and the leptonic constants, both for finite heavy quark masses and in the infinite limit are analyzed. We begin by describing the heavy meson wave equation in Sect. 2, and the interaction with the hadronic current in Sect. 3. In Sect. 4 we apply the model to the evaluation of the leptonic decay constant f_B with the aim of showing that, in the infinite heavy quark mass limit, the results of the method presented in this work coincide with those of [8]. The model has one free parameter: the mass of the shape function describing the deviation from the free propagation of one of the light quarks. This parameter is fixed in Sect. 5 by fitting the experimental results in (2). As a consequence, we are able to calculate all the form factors describing the $B \to V$ transitions. In Sect. 6 we compute the decay width $B \to \rho\ell\nu$ and compare our outcome with the results of other non perturbative approaches, as well as with the experimental data. In the Appendix we collect the relevant formulae for the various form factors describing the heavy-to-light transitions.

2 Heavy meson wave function

As discussed in the Introduction, an important aspect of the model is provided by the heavy meson wave function arising from a constituent quark picture of the heavy hadron. The heavy meson H is described as a bound state of two constituent quarks: a heavy (Q) quark and a light ($\bar{q}$) antiquark. Denoting by $\vec{k}$ the 3-momentum of the quark Q in the meson rest frame ($-\vec{k}$ is the antiquark momentum), the momentum distribution of the constituent quarks is provided by the wave function $\psi(\vec{k}) = \psi(k)$, whose Fourier transform $\Psi(\vec{r})$ is solution of the Salpeter equation [7]

$$\left[\sqrt{-\nabla^2 + m_Q^2} + \sqrt{-\nabla^2 + m_q^2} + V(r)\right] \Psi(\vec{r}) = m_H \Psi(\vec{r}) \; . \tag{4}$$

The variable r in (4) is the interquark distance, m_H is the heavy hadron mass and $V(r)$, as discussed in [8,9], is given by the Richardson potential [10]:

$$V(r) = \frac{8\pi}{33 - 2n_f} \Lambda \left(\Lambda r - \frac{f(\Lambda r)}{\Lambda r} \right) \qquad (r \geq r_m) \tag{5}$$

with the function $f(t)$ given by

$$f(t) = \frac{4}{\pi} \int_0^\infty dq \frac{sin(qt)}{q} \left[\frac{1}{ln(1+q^2)} - \frac{1}{q^2} \right] \; . \tag{6}$$

As shown by (5),(6), the Richardson potential increases linearly at large distances, thus providing confinement of the quarks, whereas at short distances it displays a Coulombic behaviour with running α_s, as dictated by perturbative QCD. For $r < r_m = \frac{4\pi\lambda}{3m_H}$ (λ is a parameter to be fitted within the model) we assume in (5) and (6) $V(r) = V(r_m)$. The reason for this cut-off of the potential is in the fact that (5), for $r \to 0$, exhibits a Coulombic divergence. Such a divergence is harmless in the nonrelativistic Schroedinger equation; on the other hand, if one of the quark masses (m_q) is light and the relativistic kinematics, embodied in the Salpeter equation, is adopted, the Coulombic divergence of the potential produces an unphysical logarithmic divergence of the wave function at the origin [11]. The form of the modified Richardson potential can be fixed by studying the problem of quark-hadron duality in $e^+e^- \to hadrons$ [9]. The potential (5) does not include spin terms, and therefore, the $J^P = 0^-, \; 1^- \; Q\overline{q}$ mesons are degenerate in mass, an approximation which is expected to work better in the limit $m_Q \to \infty$.

Notice that, due to the simple choice of the interquark potential in (5), we do not try to apply the wave equation to mesons comprising only light (u, d, s) quarks. As a matter of fact, in such a case the interaction between the quarks cannot be described by the simple form (5), since the spin terms cannot be neglected; moreover the assumption of the constituent quark picture and the instantaneous interaction is more dubious for light mesons. For example, for the description of the light pseudoscalar meson octet, the notion of light pseudoscalar particles as Nambu-Goldstone bosons has to be implemented. For this reason we describe the light mesons by using effective fields, in the spirit of the chiral effective theories and the Vector Meson Dominance Model.

For $\ell = 0$ (S-wave) heavy mesons, the only ones of interest here, (4) reduces to

$$[V(r) - m_H]\Psi(r) + \frac{2}{\pi r} \int_0^\infty dr' \Psi(r') r'$$

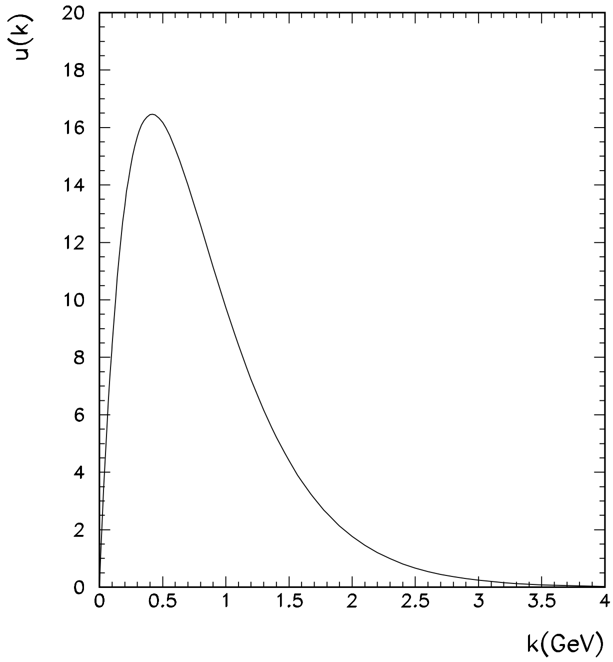

Fig. 1. The B meson reduced wave function $u(k)$

$$\times \int_0^\infty dk \left[\sqrt{k^2+m_Q^2}+\sqrt{k^2+m_q^2}\right]$$
$$\times sin(kr)\; sin(kr') \;=\; 0 \quad . \tag{7}$$

It can be solved by a numerical procedure, fitting the parameters of the model in order to reproduce the experimental meson spectrum. The following values for the parameters are obtained: $\Lambda = 397$ MeV, $\lambda = 0.6$, $m_u = m_d = 38$ MeV, $m_s = 115$ MeV, $m_c = 1452$ MeV, $m_b = 4890$ MeV. The resulting fit of the heavy meson masses can be found in [8].

The B meson reduced wave function $u(k)$ can be defined as follows:

$$u(k) = k\psi(k) \tag{8}$$

with the function

$$u(k) = \int_0^\infty r\ \Psi(r)\ sin(kr)\ dr \tag{9}$$

normalized following the relativistic prescription

$$\int \frac{d^3k}{(2\pi)^3}|\psi(k)|^2 = 2m_B\ . \tag{10}$$

An analytical representation of the B wave function, which fits the numerical solution obtained by the Multhopp method [12], is given by

$$u(k) = 4\pi\sqrt{m_B\alpha^3}\ k\ e^{-\alpha k} \tag{11}$$

with $\alpha = 2.4$ GeV^{-1}, as plotted in Fig. 1.

3 Interaction with the hadronic currents

Let us consider the matrix element:

$$< V|J^\mu|H > \tag{12}$$

where H is a heavy $(Q\bar{q})$ meson and J^μ is a hadronic current:

$$\bar{q}'\Gamma^\mu Q\ . \tag{13}$$

In (13), q' is a light quark, Γ^μ a vector combination of γ matrices and momenta and V an hadronic state not containing heavy quarks. In the sequel we shall consider only the case where H is the B meson with $J^P = 0^-$ and V = hadronic vacuum (in this case $q = q'$) or a $q'\bar{q}$ light vector meson. The formalism can be immediately extended to other cases, such as transitions between B and a light pseudoscalar meson, transitions between two heavy mesons with a current containing both heavy quarks or light quarks, etc., but we defer a detailed treatment of all these cases to a future publication.

In the constituent quark model, the straightforward approach to the evaluation of the matrix element (12) would be as follows. The heavy meson state $|H>$ is decomposed on a limited Fock base, containing only $Q\bar{q}$ pairs, with a momentum distribution weighted by $\psi(k)$. Since one knows the meson wave function only in the rest frame, the matrix element (12) has to be evaluated at rest. In general this is a limitation which allows to compute only the vacuum-single particle matrix element, while, for two single particle states, the form factors can only be evaluated at zero recoil. As a matter of fact, at zero recoil both particles are at rest and the form factors describing (12) can be extracted in the meson rest frame by working out the product of quark and antiquark operators appearing in the states $|H>$, $|V>$ and in the current (13). This procedure has been applied in [8] to the evaluation of the matrix element

$$< 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}(p) >= i f_B p^\mu \tag{14}$$

and in [13] to the calculation of the matrix element $< D^{(*)}|J^\mu|B >$ at zero recoil.

Such standard procedure cannot be immediately applied to the evaluation of the transition matrix element $B \to V$ (V is a light vector meson) for general values of the momentum transfer q^2 for several resons: 1) as discussed above, the quark constituent picture and the approximation of the instantaneous interaction are too crude for light mesons; 2) even in the approximation of the instantaneous interaction, the potential $V(r)$ in (2) and (3) is unrealistic for light mesons, since one is dropping spin terms that are not negligible for the light degrees of freedom; 3) to obtain the different form factors at various values of q^2, one would need a reliable method to boost the wave functions in a moving frame: while some recipes are available (see e.g. [14]) the prescription is not unique because our approach, similarly to all the potential models with instantaneous interaction, does not exhibit full relativistic invariance.

Instead of following this approach we propose to consider the following representation:

$$< V(p',\epsilon^*)|J^\mu|B(p) >\simeq \frac{m_V^2}{f_V}\epsilon^{*\nu}$$
$$\times \int dx\ e^{ip'x} < 0|T(V_\nu(x)J^\mu(0))|B(p) > \tag{15}$$

where $V_\nu = \bar{q}\gamma_\nu q'$ with q', q light quarks ($=u, d, s$), m_V is the vector meson mass and f_V is a coupling that can be computed by the decay widths. The approximation (15) can be seen as the implementation of the Vector Meson Dominance Model, in the limit $p'^2 \to 0$. The calculation of (15) would follow from the usual Feynman rules, with some important modifications suggested by the confined nature of the quark belonging to the heavy and light mesons. They can be summarized as follows.

1) The light meson is described by an effective field operator, in the spirit of the chiral effective field theories. To describe the light vector 1^- state, we introduce the effective operator

$$\Phi^\mu = \frac{m_V^2}{f_V}\ \overline{q'}\gamma^\mu q \ . \tag{16}$$

On a similar footing, the 0^- light meson particle (e.g. the pion) is described by the effective operator:

$$\Phi = \frac{1}{f_\pi}\overline{q'}\overleftrightarrow{\partial\!\!\!/}\ \gamma_5 q \ . \tag{17}$$

We note explicitly that in both cases the approximation is expected to work better in the limit of zero mass light mesons.

2) In the H-meson rest frame the two constituent quarks have total momentum equal to zero, whereas the sum of their energies: $E_Q + E_q = \sqrt{k^2 + m_Q^2} + \sqrt{k^2 + m_q^2}$ is different from m_H because of the presence of the interaction potential; to achieve a considerable simplification, we assume 4-momentum conservation at each hadron-quark-antiquark vertex and at the current-quark vertex. At the same time, to describe the off-shell effect, following the prescription first suggested in [15], we assume that the heavy quark has a running mass $m_Q(k)$ defined by the energy conservation equation:

$$E_Q + E_q = m_H \tag{18}$$

$$E_Q = \sqrt{k^2 + m_Q^2(k)} \tag{19}$$

$$E_q = \sqrt{k^2 + m_q^2} \ . \tag{20}$$

From previous equations, imposing $m_Q^2(k) \geq 0$, we obtain the kinematical constraint

$$0 \leq k \leq k_M = \frac{m_H^2 - m_q^2}{2m_H} \ , \tag{21}$$

i.e. $k_M = 2.64$ GeV for the $B - B^*$ system. We also note that, because of the shape of the wave function, the average value of the running mass for the B meson is $m_Q(k)_{ave} \simeq 4.6$, which is only slightly different from the mass used in the fit (4.89 GeV). We also observe that the shape of the wave function introduces an asymmetry between m_Q and m_q. In principle, we could use (18) to define the light quark mass as running mass and this would give, for the B meson, $m_q(k)_{ave} = 78$ MeV, to be compared to the result of the fit $m_q = 38$ MeV. However, in this case we would obtain that the maximum value of k is $k_M \simeq 370$ MeV and, as a consequence, the constituent quarks would be forbidden, by kinematical constraints, to reach the most likely value $(k)_{ave} \simeq 500 - 600$ MeV (see Fig. 1). For this reason one has to use (18) to define $m_Q(k)$ as the running mass.

Let us note explicitly that this procedure distinguishes between the constituent quarks, belonging to the heavy meson, that are on shell (the off-shell effects being taken into account by the running mass mechanism) and the other quarks, that we assume are able to move almost freely in the hadronic matter and will be therefore described by the free quark propagator modulated by a smooth shape function to take into account off-shell effects.

Let us now write down explicitly a set of rules for the computation of the hadronic matrix elements which implement these ideas. In order to compute a typical matrix element such as (12), a diagram like Fig. 2 can be depicted with the following correspondences.

1) For the heavy meson H in the initial state one introduces the matrix:

$$H = \frac{1}{\sqrt{3}}\psi(k)\sqrt{\frac{m_q m_Q}{m_q m_Q + q_1 \cdot q_2}}\ \frac{\not{q}_1 + m_Q}{2m_Q} \times \Gamma \frac{-\not{q}_2 + m_q}{2m_q} \tag{22}$$

where $m_Q = m_Q(k)$ is given by (18), $1/\sqrt{3}$ is a colour factor, $q_1^\mu = (E_Q, \vec{k})$, $q_2^\mu = (E_q, -\vec{k})$ are the constituent quarks momenta, with $p^\mu = q_1^\mu + q_2^\mu$ (p^μ the heavy meson momentum). Γ is a matrix which is equal to $-i\gamma_5$ for $J^P = 0^-$ and $\not{\epsilon}$ for $J^P = 1^-$. We observe that the factor $\sqrt{\frac{m_q m_Q}{m_q m_Q + q_1 \cdot q_2}}$ has been introduced to enforce the normalization condition

$$< H|H >= 2m_H \tag{23}$$

corresponding to (10). Finally, the wave function is given by (11).

2) For the heavy meson H in the final state the matrix:

$$-\gamma^0 H^\dagger \gamma^0 \ . \tag{24}$$

3) For each quark line, not representative of a constituent quark, a factor

$$\frac{i}{\not{q} - m_{q'}} \times G(q^2) \ . \tag{25}$$

As discussed above, this quark propagates almost freely in the hadronic matter. For the shape function $G(q^2)$ we assume

$$G(q^2) = \frac{m_G^2 - m_{q'}^2}{m_G^2 - q^2} \tag{26}$$

where m_G is a parameter to be fitted. Equation (26) corresponds to the Pauli-Villars regularization of the quark propagator, with mass m_G.

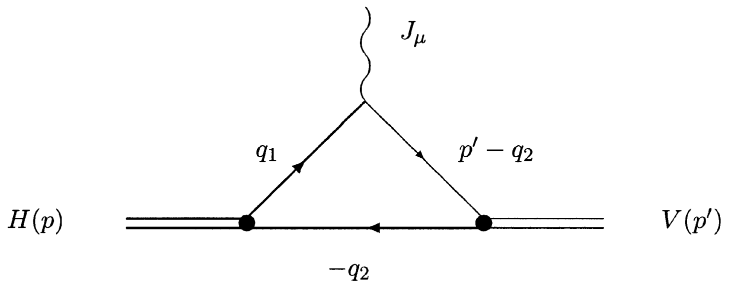

Fig. 2. Heavy lines represent constituent quarks; the light line is the (almost) free quark; $H = B, B^*$, $V = \rho, K^*$, J_μ is the current inducing the decay

4) For a light vector meson of polarization vector ϵ and quark content $q\prime$, $\overline{q}$ in the initial state the matrix

$$N_q N_{q'} \frac{m_V^2}{f_V} \not\epsilon \;, \tag{27}$$

where $f_\rho = 0.152$ GeV^2, $f_{K^*} = 0.201$ GeV^2[16]. The factor N_q is given by:

$$N_q = \sqrt{\frac{m_q}{E_q}} \quad \text{(if q = constituent quark)}$$
$$= \;\; 1 \qquad \text{(otherwise)} \;. \tag{28}$$

The reason for the factor N_q is due to a different normalization between the constituent and the (almost) free quarks.

5) For a light pseudoscalar meson M of quark content $q\prime$, $\overline{q}$, the matrix

$$N_q N_{q'} \frac{1}{f_M} (\not\ell - \not\ell') \gamma_5 \tag{29}$$

where ℓ^μ, ℓ'^μ are the quark momenta, $f_M = f_\pi \simeq 130$ MeV for pions and $f_M = f_K \simeq 160$ MeV for kaons.

5) For the hadronic current in (13) the factor

$$N_q N_{q'} \Gamma^\mu \;. \tag{30}$$

7) For each quark loop, a colour factor $N_c = 3$, a trace over Dirac matrices and an integration over k:

$$\int \frac{d^3 k}{(2\pi)^3} \theta[k_M - k] \;, \tag{31}$$

where $\theta(x)$ is the Heaviside function implementing the (18).

4 Leptonic decay constant

To compute the leptonic decay constant f_B we assume the previous rules and we immediately get, from (14),

$$f_B p^\mu = -\sqrt{3} \int \frac{d^3 k}{(2\pi)^3} \theta[k_M - k] \psi(k) \sqrt{\frac{m_q m_Q}{m_q m_Q + q_1 \cdot q_2}}$$
$$\times Tr \left[\frac{\not q_1 + m_Q}{2 m_Q} \gamma_5 \frac{- \not q_2 + m_q}{2 m_q} N_q N_Q \gamma^\mu \gamma^5 \right] . \tag{32}$$

Working out this expression in the meson rest frame we obtain

$$f_B = \frac{\sqrt{3}}{2\pi^2 m_B}$$
$$\times \int_0^{k_M} dk \; k^2 \psi(k) \frac{m_q E_Q + m_Q E_q}{\sqrt{E_q E_Q (m_q m_Q + q_1 \cdot q_2)}} \tag{33}$$

Equation (33) agrees, in the limit $m_Q \to \infty$, with the results obtained in [8] by the same model, but without the introduction of the running mass and the trace formalism. To prove the formal equivalence of the two approaches we perform the heavy quark limit in (33): $m_Q(k) \simeq m_Q(k)_{ave} \simeq m_B \gg k, m_q$, obtaining

$$f_B = \frac{\sqrt{3}}{4\pi^2 m_B} \int dk \; k^2 \psi(k) \sqrt{\frac{m_q + E_q}{E_q}}$$
$$\times \left[1 - \frac{E_q - m_q}{2 m_B} \right] \;, \tag{34}$$

which agrees with result of [8] in the same limit. Numerically, and for finite mass, the results of (33) and [8] differ, for the B meson, by 10%, which gives an estimate of the theoretical uncertainties of this procedure for the B system. In the charm case the deviations are higher (of the order $30 - 40\%$). This shows that, to apply this formalism to the $D - D^*$ system, finite heavy quark mass effects must be properly taken into account.

5 $B \to V$ form factors

Let us now apply the previous formalism to the study of the form factors describing the semileptonic decays $B \to \rho \ell \nu$ and $B \to K^* \gamma$, $B \to K^* \ell^+ \ell^-$. The corresponding matrix elements can be written as follows:

$$< V(\epsilon(\lambda), p') | \overline{q'} \gamma_\mu (1 - \gamma_5) Q | B(p) >$$
$$= \frac{2 V(q^2)}{m_B + m_V} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha p'^\beta$$
$$- i \epsilon^*_\mu (m_B + m_V) A_1(q^2)$$
$$+ i (\epsilon^* \cdot q) \frac{(p + p')_\mu}{m_B + m_V} A_2(q^2)$$
$$+ i (\epsilon^* \cdot q) \frac{2 m_V}{q^2} q_\mu [A_3(q^2) - A_0(q^2)] \;, \tag{35}$$

where

$$A_3(q^2) = \frac{m_B + m_V}{2m_V} A_1(q^2) - \frac{m_B - m_V}{2m_V} A_2(q^2) \quad , \quad (36)$$

and

$$\begin{aligned} &< V(\epsilon(\lambda), p')|\overline{q'}\sigma_{\mu\nu}q^\nu \frac{(1+\gamma_5)}{2} Q|B(p) > \\ &= 2T_1(q^2) i\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p^\alpha p'^\beta \\ &+ T_2(q^2)\left[\epsilon^*_\mu(m_B^2 - m_V^2) - (\epsilon^*\cdot p)(p+p')_\mu\right] \\ &+ T_3(q^2)(\epsilon^*\cdot p)\left[q_\mu - \frac{q^2}{m_B^2 - m_V^2}(p+p')_\mu\right] . \quad (37) \end{aligned}$$

At $q^2 = 0$ the following conditions hold

$$\begin{aligned} A_3(0) &= A_0(0) \\ T_1(0) &= T_2(0) . \quad (38) \end{aligned}$$

Let us write explicitly the matrix element of the tensor current:

$$\begin{aligned} &< V(\epsilon(\lambda), p')|\overline{q'}\sigma_{\mu\nu}q^\nu(1+\gamma_5)Q|B(p) > \\ &= \frac{N_Q N_q m_V^2}{\sqrt{3} f_V} \int \frac{d^3k}{(2\pi)^3}\theta[k_M - k]\psi(k) \\ &\times \sqrt{\frac{m_q m_Q}{m_q m_Q + q_1\cdot q_2}} G\left[(q_1 - q)^2\right] \\ &\times Tr\left[\frac{\not q_1 + m_Q}{2m_Q}\gamma_5 \frac{-\not q_2 + m_q}{2m_q}\not\epsilon^* \right. \\ &\left. \times \frac{\not q_1 - \not q + m_{q'}}{(q_1-q)^2 - m_{q'}^2}\sigma_{\mu\nu}q^\nu(1+\gamma_5)\right] . \quad (39) \end{aligned}$$

In a similar way we write all the other matrix elements. Working out the trace and performing the angular integrations we get the analytic formulae for the form factors reported in the Appendix. All these form factors depend on the shape function G defined in (26); in order to fix the unknown mass parameter m_G we consider the ratio

$$\frac{\Gamma(B\to K^*\gamma)}{\Gamma(b\to s\gamma)} = 4\left(\frac{m_B}{m_b}\right)^3\left(1 - \frac{m_{K^*}^2}{m_B^2}\right)^2 |T_1(0)|^2 . \quad (40)$$

From the experimental results (2) we obtain

$$T_1(0) = 0.19 \pm 0.05 \ , \quad (41)$$

where we have used $m_b = 4.8$ GeV. Using for $T_1(0)$ the result given in the Appendix, we obtain

$$m_G^2 \approx 3 \text{ GeV}^2 . \quad (42)$$

We can now compute, using the formulae given in the Appendix all the form factors. Their values at $q^2 = 0$ are as follows. For $B \to \rho$:

$$\begin{aligned} V(0) &= 0.45 \pm 0.11 \quad & A_2(0) &= 0.26 \pm 0.05 \\ A_1(0) &= 0.27 \pm 0.06 \quad & A_0(0) &= 0.29 \pm 0.09 . \end{aligned} \quad (43)$$

Table 1. Parameters of the various B form factors

	$F(0)$	a_F	b_F	$F(0)$	a_F	b_F	
V^ρ	0.45	1.3	0.27	0.47	1.3	0.28	V^{K^*}
A_0^ρ	0.29	1.9	1.0	0.28	1.9	0.94	$A_0^{K^*}$
A_1^ρ	0.27	0.18	0.96	0.28	0.19	0.52	$A_1^{K^*}$
A_2^ρ	0.26	1.0	1.3	0.28	0.99	0.71	$A_2^{K^*}$
T_1^ρ	0.19	1.3	0.29	0.19	1.3	0.29	$T_1^{K^*}$
T_2^ρ	0.19	0.21	1.1	0.19	0.25	0.80	$T_2^{K^*}$
T_3^ρ	0.50	1.1	0.22	0.43	0.99	0.19	$T_3^{K^*}$

For $B \to K^*$

$$\begin{aligned} T_1(0) = T_2(0) &= 0.19 \pm 0.05 \\ T_3(0) &= 0.43 \pm 0.08 . \end{aligned} \quad (44)$$

These errors are obtained by varying m_G^2 in the range $1.3 \div 7.6\ GeV^2$ corresponding to the errors in (2). In passing we observe that our results depend smoothly on the mass parameter m_G.

In Fig. 3 we report the q^2 dependence of the form factors V, A_1, A_2, A_0 T_1, T_2 and T_3 for the transitions $B \to \rho$ and $B \to K^*$.

6 Comparison with the data and other theoretical approaches

In order to give predictions on partial decay widths, we choose to fit our theoretical results for the form factors by introducing the following parameterization:

$$F(q^2) = \frac{F(0)}{1 \ - \ a_F\left(\frac{q^2}{m_B^2}\right) + \ b_F\left(\frac{q^2}{m_B^2}\right)^2} \quad (45)$$

where a_F , b_F are parameters to be fitted by means of the numerical analysis performed up to $q^2 = 15\ GeV^2$, both for ρ and K^* mesons. We collect the fitted values in Table 1.

From the table and from Fig. 3 one can see that $V(q^2)$, $T_1(q^2)$, $T_3(q^2)$ and $A_0(q^2)$ have a q^2 behaviour similar to a single pole; on the other hand, the other form factors have a practically flat behaviour. In particular, $A_1(q^2)$ shows a slight decrease. A similar behaviour is obtained by 3-point sum rules [17], but not by the light cone sum rules [18].

In Table 2 we compare our outcome for the values at $q^2 = 0$ with the results of other theoretical approaches.

We also report the predictions for the branching ratio $\mathcal{B}(\bar B^0 \to \rho^+\ell\nu)$ and for the partial widths at fixed helicity:

$$\mathcal{B}(\bar B^0 \to \rho^+\ell\nu) = 2.4 \times 10^{-4} \quad (46)$$

$$\begin{aligned} \Gamma_0 &= 2.4 \times 10^{-17}\ s^{-1} \\ \Gamma_+ &= 4.6 \times 10^{-18}\ s^{-1} \\ \Gamma_- &= 7.4 \times 10^{-17}\ s^{-1} \end{aligned} \quad (47)$$

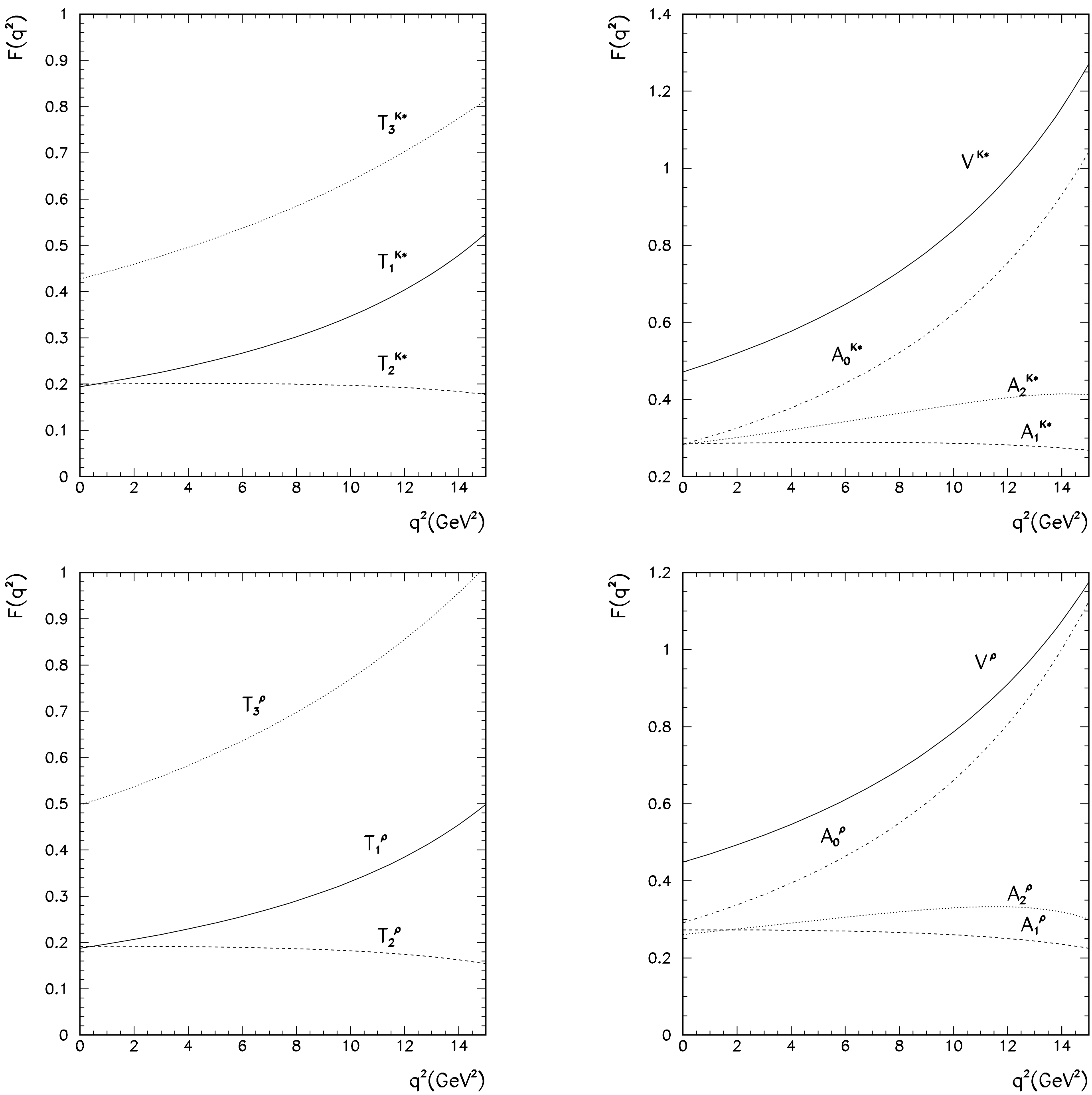

Fig. 3. q^2 behaviour of semileptonic and rare B form factors. From left to right and from up to down: $B \to K^*$ (rare), $B \to K^*$ (semileptonic), $B \to \rho$ (rare), $B \to \rho$ (semileptonic)

where Γ_0, Γ_+, Γ_- refer to the ρ helicities. One can see that there is agreement between the result (46) and the experimetal data in (1).

In conclusion, the calculation based on the present QCD relativistic quark model seems quite adequate to describe the weak transition $B \to$ light vector meson. In spite of its simplicity the model embodies many of the features of more fundamental approaches; in particular it is confining and it contains the perturbative QCD α_s corrections through the Coulombic behaviour of the potential at small distances. Therefore it can be seen as a rather realistic model of the fundamental QCD description of these important weak processes.

Table 2. Comparison of the results coming from different works on form factors

	This work	*LCSR* [18]	*LCSR* [19]	*LCSR* [20]	*SR* [17]	*Latt.* + *LCSR*[21]
$V^\rho(0)$	0.45 ± 0.11	0.34 ± 0.05	0.35 ± 0.07	0.37 ± 0.07	0.6 ± 0.2	$0.35^{+0.06}_{-0.05}$
$A_0^\rho(0)$	0.29 ± 0.09				0.24 ± 0.02	$0.30^{+0.06}_{-0.04}$
$A_1^\rho(0)$	0.27 ± 0.06	0.26 ± 0.04	0.27 ± 0.05	0.30 ± 0.05	0.5 ± 0.1	$0.27^{+0.05}_{-0.04}$
$A_2^\rho(0)$	0.26 ± 0.05	0.22 ± 0.03	0.28 ± 0.05	0.33 ± 0.05	0.4 ± 0.2	$0.26^{+0.05}_{-0.03}$
$T_1^\rho(0)$	0.19 ± 0.05	0.15 ± 0.02	0.12 ± 0.04	0.15 ± 0.05		
$T_3^\rho(0)$	0.50 ± 0.08	0.10 ± 0.02		0.10 ± 0.05		
$V^{K^*}(0)$	0.47 ± 0.11	0.46 ± 0.07	0.38 ± 0.08	0.45 ± 0.08	0.47 ± 0.03	
$A_0^{K^*}(0)$	0.28 ± 0.09				0.30 ± 0.03	
$A_1^{K^*}(0)$	0.28 ± 0.07	0.34 ± 0.05	0.32 ± 0.06	0.36 ± 0.05	0.37 ± 0.03	
$A_2^{K^*}(0)$	0.28 ± 0.05	0.28 ± 0.04		0.40 ± 0.05	0.40 ± 0.03	
$T_1^{K^*}(0)$	0.19 ± 0.05	0.19 ± 0.03	0.16 ± 0.03	0.17 ± 0.05	0.19 ± 0.03	$0.16^{+0.02}_{-0.01}$
$T_3^{K^*}(0)$	0.43 ± 0.08	0.13 ± 0.02		0.13 ± 0.05	0.3	

A Form factors

In this Appendix we report the expressions of the various form factors for the weak transitions $B(p) \to V(p', \epsilon)\ell\nu$. We note that, according to the discussion after 15, we put $p'^2 = 0$.

$$
\begin{aligned}
V(q^2,x) &= \frac{m_B + m_V}{m_B}\,\frac{m_V^2}{f_V}\frac{\sqrt{3}}{8\pi^2} \\
&\times \int_0^{k_M} \frac{dk\; u(k)}{\sqrt{E_Q E_q (m_Q m_q + E_Q E_q + k^2)}} \\
&\times \frac{1}{\left(1 - \frac{q^2}{m_B^2}\right)} \\
&\times \left\{ \frac{2k}{|\vec{q}|}\;\frac{(m_Q - m_{q'})m_B - (m_Q - m_q)q^0}{m_B} \right. \qquad (48)\\
&+ \left[\frac{m_q E_Q + m_Q E_q}{m_B} \right. \\
&- \frac{(m_Q - m_{q'})m_B - (m_Q - m_q)q^0}{m_B} \\
&\times \left.\left. \frac{2|\vec{q}|E_q - m_q^2 + x^2}{2|\vec{q}|^2} \right] \ln g(q^2,k,x) \right\}
\end{aligned}
$$

$$
\begin{aligned}
A_1(q^2,x) &= \frac{1}{(m_B + m_V)m_B}\;\frac{m_V^2}{f_V}\frac{\sqrt{3}}{4\pi^2} \\
&\times \int_0^{k_M} \frac{dk\; u(k)}{\sqrt{E_Q E_q (m_Q m_q + E_Q E_q + k^2)}} \\
&\times \frac{1}{\left(1 - \frac{q^2}{m_B^2}\right)} \\
&\times \left\{ -(m_Q - m_q)\;\frac{k}{|\vec{q}|}\;(2|\vec{q}|E_q - m_q^2 + x^2) \right. \\
&+ 2(m_Q - m_q)k\,|\vec{q}| \qquad (49)\\
&+ \left[-q^0(m_Q E_q + m_q E_Q) \right. \\
&+ (m_{q'} + m_Q)(m_B E_Q - m_Q^2 + m_Q m_q) \\
&- (m_Q - m_q)\frac{2|\vec{q}|E_q - m_q^2 + x^2}{2} + k^2(m_Q - m_q) \\
&\times \left.\left. \left(\frac{(2|\vec{q}|E_q - m_q^2 + x^2)^2}{4|\vec{q}|^2 k^2} - 1 \right) \right] \ln g(q^2,k,x) \right\}
\end{aligned}
$$

$$
\begin{aligned}
A_2(q^2,x) &= -\frac{(m_B + m_V)}{m_B}\;\frac{m_V^2}{f_V}\frac{\sqrt{3}}{4\pi^2} \\
&\times \int_0^{k_M} \frac{dk\; u(k)}{\sqrt{E_Q E_q (m_Q m_q + E_Q E_q + k^2)}} \\
&\times \frac{1}{\left(1 - \frac{q^2}{m_B^2}\right)} \left\{ \frac{k}{|\vec{q}|} \left[m_{q'} + m_Q \right.\right. \\
&+ \left. \frac{q^0}{m_B}(m_q - 3m_Q) + \frac{2(m_Q - m_q)E_Q q^2}{m_B^3} \right] \\
&- \frac{2|\vec{q}|E_q - m_q^2 + x^2}{2m_B^2|\vec{q}|^2} \\
&\times (m_Q - m_q)(2|\vec{q}| - 3m_B)k \qquad (50)\\
&+ \left[\frac{m_Q - m_q}{2m_B|\vec{q}|}k^2 - \frac{E_Q - E_q}{2m_B^2}(m_q E_Q + m_Q E_q) \right. \\
&+ \frac{m_Q - m_q}{m_B^2}(2|\vec{q}| - 3m_B)\frac{(2|\vec{q}|E_q - m_q^2 + x^2)^2}{8|\vec{q}|^3} \\
&- \frac{2|\vec{q}|E_q - m_q^2 + x^2}{4|\vec{q}|^2} \left(m_{q'} + m_Q \right. \\
&+ \left.\left. \frac{q^0}{m_B}(m_q - 3m_Q) + \frac{2(m_Q - m_q)E_Q q^2}{m_B^3} \right) \right]
\end{aligned}
$$

$$\times \ln g(q^2,k,x)\Bigg\}$$

$$\begin{aligned}
A_0(q^2,x) &= \frac{m_V}{f_V}\frac{\sqrt{3}}{8\pi^2 m_B} \\
&\times \int_0^{k_M} \frac{dk\ u(k)}{\sqrt{E_Q E_q (m_Q m_q + E_Q E_q + k^2)}} \\
&\times \frac{1}{\left(1-\frac{q^2}{m_B^2}\right)}\Bigg\{ -2k\,|\vec{q}| \\
&\times \Bigg[\frac{2(m_Q-m_q)E_Q}{m_B^2|\vec{q}|}q^2 - \frac{1}{m_B|\vec{q}|} \\
&\times \Bigg((m_Q+m_{q'})m_B q^0 + (m_Q-m_q)q^2 \Bigg) \\
&- m_{q'} + m_Q \Bigg] \\
&+ \frac{2k\ (m_Q-m_q)}{m_B}(2|\vec{q}|E_q - m_q^2 + x^2) \qquad (51)\\
&+ \Bigg[m_Q(m_q-m_Q)(m_Q+m_{q'}) + m_Q q^2 \\
&- \frac{m_Q-m_q}{m_B}\frac{(2|\vec{q}|E_q - m_q^2+x^2)^2}{2|\vec{q}|} \\
&+ \frac{E_Q}{m_B}\Bigg(-2m_B m_Q q^0 \\
&+ (m_Q-m_q)(2q^0E_Q - q^2) + m_B^2(m_Q+m_{q'}) \Bigg) \\
&+ \Bigg(\frac{2(m_Q-m_q)E_Q}{m_B^2|\vec{q}|}q^2 \\
&- \frac{1}{m_B|\vec{q}|}\Big((m_Q+m_{q'})m_B q^0 + (m_Q-m_q)q^2 \Big) \\
&- m_{q'} + m_Q \Bigg) \Bigg] \\
&\times \frac{2|\vec{q}|E_q - m_q^2 + x^2}{2} \ln g(q^2,k,x) \Bigg\} .
\end{aligned}$$

Here:

$$|\vec{q}| = \frac{m_B^2 - q^2}{2} \qquad q^0 = \sqrt{q^2 + |\vec{q}|^2} \qquad (52)$$

$$g(q^2,k,x) = \frac{|2k|\vec{q}| + 2|\vec{q}|E_q - m_q^2 + x^2|}{|-2k|\vec{q}| + 2|\vec{q}|E_q - m_q^2 + x^2|} . \qquad (53)$$

The results for the form factors describing the decay $B(p) \to V(p',\epsilon)\gamma$ are as follows ($p'^2 = 0$):

$$\begin{aligned}
T_1(q^2,x) &= \frac{m_V^2}{f_V}\frac{\sqrt{3}}{16\pi^2 m_B} \\
&\times \int_0^{k_M} \frac{dk\ u(k)}{\sqrt{E_b E_q (m_b m_q + E_b E_q + k^2)}} \frac{1}{\left(1-\frac{q^2}{m_B^2}\right)} \\
&\times \Bigg\{ -2\frac{k}{|\vec{q}|}\Bigg[q^2 - 2E_b q^0 + \frac{q^0}{m_B}(m_b+m_{q'}) \\
&\times (m_b - m_q) \Bigg] - \frac{k}{|\vec{q}|}(2|\vec{q}|E_q - m_q^2 + x^2) \\
&+ \Bigg[(m_b+m_{q'})\frac{m_bE_q + m_qE_b}{m_B} + k^2 \qquad (54)\\
&+ \frac{(2|\vec{q}|E_q - m_q^2+x^2)^2}{4|\vec{q}|^2} + \frac{2|\vec{q}|E_q - m_q^2 + x^2}{2} \\
&\times \Bigg(\frac{(m_b+m_{q'})(m_b-m_q)q^0}{m_B|\vec{q}|^2} + \frac{q^2 - 2E_bq^0}{|\vec{q}|^2} \Bigg) \Bigg] \\
&\times \ln g(q^2,k,x) \Bigg\}
\end{aligned}$$

$$\begin{aligned}
T_2(q^2,x) &= \frac{1}{m_B^2 - m_V^2}\frac{m_V^2}{f_V}\frac{\sqrt{3}}{8\pi^2 m_B} \\
&\times \int_0^{k_M} \frac{dk\ u(k)}{\sqrt{E_b E_q (m_b m_q + E_b E_q + k^2)}} \frac{1}{\left(1-\frac{q^2}{m_B^2}\right)} \\
&\times \Bigg\{ 2k\,|\vec{q}|[(m_b+m_{q'})(m_q-m_b) + 2m_BE_b] \\
&- m_B q^0 k \frac{2|\vec{q}|E_q - m_q^2 + x^2}{|\vec{q}|} \\
&+ \Bigg[q^0[m_B m_b m_{q'} + E_b(m_b+m_{q'})(m_q-m_b) \\
&+ m_B E_b^2] + q^2[m_b(m_b-m_q) - m_BE_b] \\
&- [(m_b+m_{q'})(m_q-m_b) + 2m_BE_b] \\
&\times \frac{2|\vec{q}|E_q - m_q^2 + x^2}{2} \qquad (55)\\
&+ m_B q^0 \frac{(2|\vec{q}|E_q - m_q^2 + x^2)^2}{4|\vec{q}|^2} \Bigg] \ln g(q^2,k,x) \Bigg\}
\end{aligned}$$

$$\begin{aligned}
T_3(q^2,x) &= \frac{m_V^2}{f_V}\frac{\sqrt{3}}{8\pi^2 m_B} \\
&\times \int_0^{k_M} \frac{dk\ u(k)}{\sqrt{E_b E_q (m_b m_q + E_b E_q + k^2)}} \frac{1}{\left(1-\frac{q^2}{m_B^2}\right)} \\
&\times \Bigg\{ -k\Bigg[-2E_b + \frac{1}{|\vec{q}|}\Bigg(-m_B^2 + \frac{2m_B - q^0}{m_B} \\
&\times (m_b+m_{q'})(m_b-m_q) \Bigg) \\
&+ \frac{2|\vec{q}| + 3q^0}{2|\vec{q}|^2}(2|\vec{q}|E_q - m_q^2 + x^2) \Bigg] \qquad (56)
\end{aligned}$$

$$+ \left[\frac{m_b(m_b - m_{q'})}{2} + m_b(m_q - m_b) + m_B E_q \right.$$
$$+ \frac{E_b}{m_B} \frac{(m_b + m_{q'})(m_b - m_q)}{2} - \frac{q^0 k^2}{2|\vec{q}|}$$
$$+ \frac{2|\vec{q}|E_q - m_q^2 + x^2}{2|\vec{q}|} \Bigg(- E_b + \frac{1}{2|\vec{q}|}\Big(- m_B^2$$
$$+ \frac{2m_B - q^0}{m_B}(m_b + m_{q'})(m_b - m_q)\Big)\Bigg)$$
$$\left. + \frac{2|\vec{q}| + 3q^0}{8|\vec{q}|^3}(2|\vec{q}|E_q - m_q^2 + x^2)^2 \right] \ln g(q^2, k, x) \Bigg\}$$

All the form factors are obtaind by taking the difference $F(q^2) = F(q^2, x = m_{q'}) - F(q^2, x = m_G)$, where m_G is the mass parameter defined in the text; moreover $m_{q'} = m_q$ when $V = \rho$, while $m_{q'} = m_s$ when $V = K^*$.

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